Classification of parallelisms of projective spaces

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❖ Description of the problem

Parallelisms are partitions of the lines of a projective space to spreads, where each point is in exactly one line of each spread. They are intensively studied because of their relation to translation planes, combinatorial designs, network coding, error-correcting codes, and cryptography. Computer-aided constructive classifications of parallelisms with given relatively small parameters and assumed automorphism groups are of particular interest both for possible applications and for future theoretical investigations, and there are already many authors and results in this field. The classification algorithms, however, are based on backtrack search and are exponential. That is why the access to powerful computers is of major importance when bigger parameters are considered.

❖ Use of HPC Infrastructure

For our parallel implementation of the backtrack search for parallelisms we use High-performance C++ Intel Parallel Studio XE which enables our MPI based C++ software to take advantage of multi-core processors [1].

Our software does not imply any communication among the different processes. It performs backtrack search with rejection of equivalent partial solutions at several stages We input a file with the partial parallelism which has to be extended to parallelisms and additional files with the line and spread orbits under the action of the considered group. Each process starts extending the partial parallelism up to partial parallelisms with a given size Z. Each process extends only some of the partial solutions of size Z and writes the results in its own file. The efficiency of this algorithm depends on the choice of Z.

The results are obtained using up to 6 HP Cluster Platform SL250S GEN8 servers, each with 2 Intel Xeon E2650 v2 CPUs on the supercomputer Avitohol [1].

❖ Results

We have applied this parallel algorithm to the two classification problems presented below:

1. Subregular parallelisms of PG(3,4) with an automorphism group of order 2 [2].

Automorphism group	Subregular Parallelisms of PG(3,4) from 8268 partial parallelisms
2	7999
4	244
5	346
7	24
10	2
20	8
All	8623

2. Parallelisms of PG(3,5) invariant under a cyclic automorphism group of order 8 [3].

Automorphism group	Parallelisms of PG(3,5) from 106 partial parallelisms
8	8143
16	952
24	610
32	56
48	90
96	6
1200	4
2400	2
All	9863

In the first case several hours on a 3GHz PC are needed for the extension of one partial parallelism and in the second – several days. The number of partial solutions which we process and the number of the obtained parallelisms can be seen in the tables above and shows that the investigations became possible due to the usage of the high-performance computing system Avitohol.

1. http://nchdc.acad.bg/en/resources/iict/avitohol/

2. Betten A., Topalova S., Zhelezova S.:New Uniform Subregular Parallelisms of PG(3,4) Invariant under an Automorphism of Order 2.

3. Topalova S., Zhelezova S.: New Parallelisms of PG(3,5) with Automorphisms of Order 8, CASC, 2021.